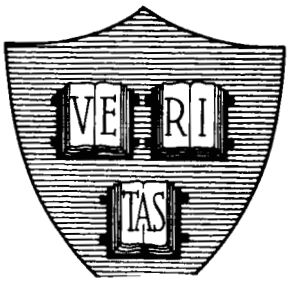
Division of Engineering and Applied Physics
Harvard University Cambridge, Massachusetts

# SEMIANNUAL PROGRESS REPORT No. 70






## Covering Period

## January 1, 1965 – July 1, 1965

November, 1965

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November 1965

Division of Engineering and Applied Physics

Harvard University

Cambridge, Massachusetts

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| Nonr-1866(16) | The Steering Committee |
| Nonr-1866(32) | R. W. P. King |

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| DA-ARO-D-31-124-G697 | N. Bloembergen |
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SPR70 Joint Services Electronics Program

January 1, 1965 - July 1, 1965

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# I. VERY LOW FREQUENCY PROPAGATION

J. A. Pierce

## 1. Navigation

Contract Nonr-1866(07).

Mr. Pierce continues to be an active member of the Omega Implementation Committee, and the work toward the system itself is accelerating. Aside from many meetings and conferences, and a trip to Trinidad to examine a possible site for a future Omega station, the Harvard contribution has consisted largely in studies of the stability of the group velocity of propagation. These results are discussed below under the heading "Transmission Time." They are of great importance in navigation because they determine the probability of successful land identification, which is a problem of the first order, if circumstances do not permit the navigator to maintain continuous tracking of his signals.

Considerable effort has also been expended in calculating the field strength and phase of signals received at each Omega station in a number of proposed operational configurations. These calculations are complex because, as explained in earlier reports, the phase and amplitude of the signals depend upon time, direction, latitude, and soil conductivity. If the stations are to maintain satisfactory phase relationships it is important that they be able to observe each other for several hours per day with a minimum of interference from unwanted modes of transmission. The choice of each station site depends upon all other sites, and the selection of the remainder, after two or three have been chosen, becomes relatively difficult.

This is, in an operational system, a requirement for a small amount of communication between stations, preferably requiring no relaying of in-

formation. For this service it is of no importance whether the signals are propagated by one mode or another, or by short-way or long-way transmission around the earth. In a network of eight stations, our studies indicate that this overall requirement cannot always be met. A few of the required fifty-six transmission paths seem likely to suffer from destructive interference between paths for several hours a day. In these cases, it will be necessary to relay information or to wait until transmission is satisfactory.

## 2. Regulation of Oscillator Frequency

A variant of the standard method of referring a local oscillator to a VLF signal has been put into use in this report period. A phase detector and servo system of excellent accuracy are used to track the 20 kc/s signal from WWVL at Fort Collins, Colorado, and to control, through a simple aided-tracking loop, a crystal oscillator having a very uniform aging characteristic. The aging is compensated by precessing the dial of the oscillator at the predicted rate, and the phase information is fed into the oscillator frequency with about a 30-hour time constant. The time constant of the phase-detection loop is about 15 minutes.

The servo tracking is gated by a clock so that it is active only during the daylight hours. The phase-shifter output is thus an accurate reproduction of the WWVL received phase in the daytime while at night the output is that of a crystal which has been accurately adjusted to agree with the WWVL frequency. The stability of the oscillator is indicated by the fact that in the morning a correction of more than a microsecond is seldom needed.

By this mechanism we have achieved a frequency source that is demonstrably in agreement with WWVL to about a part in $10^{11}$ at all times, while

agreement for day-to-day observations in the sunlit hours is even greater. By using this frequency source as one of the inputs to our frequency combiner, previously described, it is now possible to refer the frequencies of our other sources to the national standard with great ease.

Study of the steering techniques for maintenance of a local frequency source in nearly constant phase with respect to a distant standard is being continued. An example of the results of this method, which was described in Progress Report No. 69, is shown in Fig. 1. The over-correction of frequency, to cause the phase to return to its nominal value, is easily seen, as are several instances of oscillator vagaries lasting a few days before returning to "normal." An especially obvious example is the sudden increase in frequency about 23 August followed by a decrease of about the same amount on 30 August.

The precise details of this steering technique depend upon the magnitude of the phase noise on the transmission path and on the stability of the frequency sources. However, Fig. 1 and similar studies indicate that a relative phase can be maintained to good accuracy over indefinitely long periods.

### 3. Transmission Time

The need for practical decisions about the techniques for lane identification in Omega are forcing us to draw conclusions somewhat in advance for the collection of adequate experimental data. In this dilemma, we are forced to guess at the way in which the standard deviation of the phase of a modulation is related to the standard deviation of a carrier. An example of such estimation, that "feels" as though it may be a reasonable approximation to the truth, is given below, together with some estimates of the reliability of Omega lane identification that may be derived from it.

The time of transmission, $t_m$, of a modulation or a difference frequency is

$$t_m = \frac{f_2 t_2 - f_1 t_1}{f_2 - f_1} \tag{1}$$

where $t_1$ and $t_2$ are the transmission times of the lower and higher frequency components and $f_1$ and $f_2$ are the lower and higher frequencies.

It follows that the standard deviation, $\sigma_m$, of the time of arrival of a particular phase of the modulation is related to the standard deviations of the carrier components by

$$\sigma_m^2 = \frac{f_1^2 \sigma_1^2 - 2r f_1 f_2 \sigma_1 + f_2^2 \sigma_2^2}{(f_2 - f_1)^2} \tag{2}$$

where $\sigma_1$ and $\sigma_2$ are the standard deviations of the lower and higher frequency components, and r is the correlation coefficient relating them.

If $\sigma_1 = \sigma_2 = \sigma_c$,

$$\sigma_m^2 = \frac{f_1^2 - 2r f_1 f_2 + f_2^2}{(f_2 - f_1)^2} \sigma_c^2 \tag{3}$$

By substituting the identity

$$2f_1 f_2 + (f_2 - f_1)^2 = f_1^2 + f_2^2 \tag{4}$$

in Equation (3), we obtain

$$\frac{\sigma_m^2}{\sigma_c^2} = \frac{2f_1 f_2 - 2r f_1 f_2 + (f_2 - f_1)^2}{(f_2 - f_1)^2}$$

$$= \frac{2f_1 f_2 (1 - r)}{(f_2 - f_1)^2} + 1 \tag{5}$$

Let us define M as the approximate magnification ratio for deviations in the absence of correlation, or

$$M = \frac{(2f_1 f_2)^{\frac{1}{2}}}{f_2 - f_1} \tag{6}$$

Then

$$\frac{\sigma_m^2}{\sigma_c^2} = M^2(1-r) + 1 \tag{7}$$

It is difficult to measure r experimentally, because

$$1-r \approx \frac{\sigma_m^2}{M^2 \sigma_c^2} \tag{8}$$

Conversely, since $\sigma_m$ varies as $(1-r)^{\frac{1}{2}}$, the effects of correlation will not be experimentally conspicuous unless the coefficient approaches + 1.

It is reasonable to assume that, in a limited frequency region, the departure from perfect correlation is proportional to the relative frequency separation, or

$$1-r = \frac{k^2(f_2 - f_1)}{(2f_1 f_2)^{\frac{1}{2}}} = \frac{k^2}{M} \tag{9}$$

where k is a dimensionless constant of proportionality.

Combining this assumption with Equation (7),

$$\frac{\sigma_m^2}{\sigma_c^2} = Mk^2 + 1 \tag{10}$$

or

$$\sigma_m^2 = Mk^2\sigma_c^2 + \sigma_c^2 \qquad (11)$$

In the cases of interest for Omega, M ranges from 5 to several hundred, and k appears to be of the order of 1 or 2. The last term in Equation (11) is therefore unimportant and, to a sufficient approximation,

$$\sigma_m = k\sigma_c M^{\frac{1}{2}} \qquad (12)$$

We have, in connection with Equation (3), assumed that $\sigma_c$ is invariant with frequency, although there are indications that it decreases at higher frequencies. We may, therefore, try considering $k\sigma_c$ as a constant. We have two measured values of $\sigma_m$ (each averaged over the entire day) at frequency differences of 3400 (13600-10200) c/s and 425 (13600 modulated by $212\frac{1}{2}$) c/s, as follows:

| $f_2-f_1$ | $M^{\frac{1}{2}}$ | Observed $\sigma_m$ in cels* | $k\sigma_c$ in cels |
|---|---|---|---|
| 3400 | 2.21 | 12 | 5.43 |
| 425 | 6.73 | 36 | 5.36 |

We may therefore presume that

$$\sigma_m = 5.4\, M^{\frac{1}{2}} \text{ cels} \qquad (13)$$

subject to later delineation of the correlation coefficient and determination of the variation of the standard deviation of the carrier as a function of frequency.

Conditions (such as night-time transmission) that induce larger

---

* A "cel" is 1/100 of a period at 10200 c/s.

standard deviations of the carrier may also be characterized by greater correlation and smaller values of k. If this be the case, the product $k\sigma_c$ may be more truly a constant than either multiplier alone, and the standard deviation of the modulation may have less relative diurnal variation than that of the carrier.

We can, with the aid of Equation (13), estimate the probability of success in the various lane identifications proposed for Omega. Two preliminary steps must be taken,however. Equation (13) makes no allowance for difficulties in predicting the diurnal variation of the modulation phase with respect to the carrier phase. It may be presumed that eventually this variation can be predicted to about half a standard deviation, as has been done in the case of the spatial and temporal variations of the carrier phase at 10200 c/s. For the present, however, let us assume that the prediction error is equal to the standard deviation. This assumption requires the increase of the constant in Equation (13) by $(2)^{\frac{1}{2}}$, making it 7.6 cels.

It is also necessary to account for instrumental errors in the determination and comparison of the various phases. It may be assumed that care enough will be taken and that a sufficiently long time-constant will be used to give each necessary measurement a standard deviation of one percent of the period. These errors are, of course, not correlated with the propagational errors. We can thus modify Equation (13) to read

$$\sigma_{mod.} = \left[(7.6\ M)^2 + (10^{-2}P)^2\right]^{\frac{1}{2}} \text{ in cels} \qquad (14)$$

where P is the period of the modulation in cels.

The probability of successful identification can be assessed by dividing the half-period of the frequency to be identified by the standard

deviation at the identifying frequency. This calculation can be made in three ways:

1. Identification signal-by-signal; done by computing the probability of deviations less than $n\sigma$, where n is the ratio defined above.
2. Identification pair-by-pair; in which the standard deviations of Case 1 must be multiplied by $(2)^{\frac{1}{2}}$.
3. Identification fix-by-fix; in which the probabilities of Case 2 are squared, to exclude failures in either coordinate of the fix, or both.

Table I (on page 6) exhibits some characteristics of the various identifying frequencies and the more important probabilities for the three kinds of identification defined above.

It should be noted that identification signal-by-signal, although having the greatest probability of success, requires calibration of the various phase shifts to be expected in a receiver. The other cases avoid this difficulty, and are likely to be preferred for this reason, provided that the actually-achieved reliability of identification is high enough for operational use.

It is recognized that $\sigma_c$ is greater ( and k possibly smaller) by night than by day. The probabilities computed in Table I are therefore optimistic for night and pessimistic for daytime operation. The true calculation of the diurnal values of the probabilities must await the collection of more extensive data than are yet available.

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TABLE I

Probabilities of Successful Lane Identification

| Identifying Frequency in c/s | M | Period in cels | Estimated $\sigma_{mod.}$ in cels | Probabilities Case I Signal | Case 2 Pair | Case 3 Fix |
|---|---|---|---|---|---|---|
| | To identify 1200 c/s: $\frac{1}{2}$ period = 50 cels | | | | | |
| 3400 | 4.90 | 300 | 17.1 | 0.997 | 0.96 | 0.92 |
| | To identify 3400 c/s: $\frac{1}{2}$ period = 150 cels | | | | | |
| $1133\frac{1}{3}$ | 13.4 | 900 | 29.3 | 0.999+ | 0.999+ | 0.999 |
| | To identify 1133 $\frac{1}{3}$ c/s. : $\frac{1}{2}$ period = 450 cels | | | | | |
| $226\frac{2}{3}$ | 42.4 | 4500 | 67.0 | 0.999+ | 0.999+ | 0.999+ |
| | To identify 226 $\frac{2}{3}$ c/s. : $\frac{1}{2}$ period = 2250 cels | | | | | |
| $45\frac{1}{3}$ | 177 | 22500 | 247 | 0.999+ | 0.999+ | 0.999+ |
| | To identify 45 $\frac{1}{3}$ c/s. : $\frac{1}{2}$ period = 11250 | | | | | |
| $11\frac{1}{3}$ | 636 | 90000 | 920 | 0.999+ | 0.999+ | 0.999+ |

SPR70 II. ELECTRON AND SOLID STATE PHYSICS

## Personnel

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Mr. P. Lallemand
Mr. C. H. Lee
Mr. L. Malmstrom
Mr. J. L. Merz

### 1. Maser Spectrometer, S. Dmitrevsky. Contract Nonr-1866(16)

The observed behavior of a maser oscillator spectrometer has shown that the oscillator noise is the limiting feature of the device performance. A series of experiments was therefore undertaken to investigate the noise in the light of recent theoretical studies.

It has been established that there exists a peak (corresponding to two sidebands of the signal frequency) in the maser noise spectrum centered about the frequency corresponding to the frequency of the transient ripple of the maser output envelope. The amplitude of the power spectrum and the shape of the peak has been observed to depend on various parameters of the oscillator (loaded Q, pump power, number of photons in the cavity) in a manner which substantiates the point of view that the noise is due to spontaneous emissions of both the signal frequency photons and pump frequency phonons, each emission exciting appropriate damped oscillatory transients (as described and predicted by Statz - de Mars equations and their variants) of the output envelope. The combined effect of these elementary excitations as observed conforms to the results predicted by using Campbell's Theorem of the shot effect theory.

The above results justify theoretical arguments explaining maser oscillator noise as a form of the shot effect, with photons of the stored signal energy and various level populations being the relevant discrete entities.

- - - - -

2. Electric Field Effect in Endor Spectroscopy, J. F. Reichert.

Contract Nonr-1866(16).

The Endor spectrometer has been modified to facilitate signal averaging techniques. The spectrometer has been applied to a study of the effects of electric field on the Endor spectrum of F-centers in KCl. Preliminary analysis of the data tends to confirm theoretical predictions about the effect. A complete analysis of the data is in progress and a report of the results obtained so far is being prepared. This will be available for the next reporting period.

- - - - -

3. Nonlinear Optical Properties of III-V Compounds, R. K. Chang.

Contract Nonr-1866(16).

Measurements of the dispersion of the nonlinear susceptibility of III-V compounds have been continued. In total, the nonlinear susceptibility of four semiconductors with $\bar{4}3$ symmetry was measured at nine different wavelengths covering the range from 1.06 micron to .53 micron. Theoretical work was also done in order to interpret the observed structure in the dispersion curve.

The phase of the complex nonlinear susceptibility was measured for the first time during this reporting period. The real and imaginary parts of the nonlinear susceptibility are now completely determined. Accounts have been published in Physical Review Letters 75, 6 (1965) and 15, 415 (1965).

- - - - -

4. Inverse Faraday Effect, L. D. Malmstrom, J. P. van der Ziel and P. S. Pershan. Contract Nonr-1866(16).

The inverse Faraday effect (IFE) has now been observed in $Eu^{++}$: $CaF_2$, a number of liquids and a number of diamagnetic glasses. In all of these materials, the magnetization was found strictly proportional to the Verdet constants, the proportionality constant being independent of the material. In Europium doped $CaF_2$, the Verdet constant and the induced magnetization were both found proportional to $T^{-1}$. The induced magnetization in all of these materials was also found linearly proportional to the intensity of circularly polarized light.

The results of this research were reported at the Physics of Quantum Electronics Conference, San Juan, Puerto Rico, June, 1965. A preliminary account of these results has also been published in Physical Review Letters [1].

- - - - -

5. Studies on the Stimulated Raman Effect, P. Lallemand. Contract Nonr-1866(16).

The properties of a light amplifier at the Raman frequency have been studied. Experimentally, the laser beam generates a Stokes beam in a first oscillator cell; then these two beams interact in a second cell, a 30 db attenuator at the Stokes frequency being used as an isolator. We find that the gain in the second cell, as a function of length, is the sum of two exponential terms. For one, the gain constant is in accordance with the spontaneous Raman cross section; for the other, which dominates when the two cells are close to one another, the gain constant is about 30 times larger. That high gain is proportional to the laser power, and decreases when the input Stokes power increases.

Using such an amplifier, we have studied the tensorial properties of the Raman nonlinear susceptibility. The values found for the depolarization factor, using a linearly polarized laser beam, or a circularly polarized beam are consistent with Placzek's polarizability theory; although the actual value is about twice the value obtained through the spontaneous Raman effect. This may be due to gradients of index or refraction in the Raman medium coming from electrostrictive effects or real parts of the nonlinear susceptibility.

We have studied some liquid mixtures. In benzen - acetone, the gain at the Stokes frequency of benzene is proportional to the concentration of benzene. In benzene-nitrobenzene, the gain remains constant until about 50 % and then drops to a low value, but different from zero, in pure nitrobenzene.

The main results of this work have been reported:
P. Lallemand and N. Bloembergen: Multimode Effects in Stimulated Raman Effect, Applied Physics Letters 6, 210; 212 (1965). N. Bloembergen and P. Lallemand: Light Waves with Exponential Gain, Physics of Quantum Electronics Conference, Puerto Rico, June, 1965.

6. Charge Conversion of Trivalent Rare Earth Ions in $CaF_2$, J. L. Merz. Contract Nonr-1866(16).

Improved instrumentation made possible a highly reproducible series of measurements on a temperature dependence of the thermal luminescence between 80° and 460°K. Using 50 kv, 20 ma X-rays, it was found that the temperature dependence of the thermal luminescence was independent of dosage up to from 30 minutes to 1 hour. For longer irradiation, saturation effects set in. Concentration-dependent studies between 0.1% and 0.003% indicate that there are at least two types of centers involved in the thermo-

luminescence process. Low temperature glow peaks are not influenced appreciably by concentration while high temperature glow peaks vary widely.

Further studies have been made on Er, Sm, Dy, Ho, Tm, Pr, Gd, and Yb. In most cases the temperature dependence of the low temperature glow peaks are very similar for each of these ions in calcium fluoride. The high temperature glow peaks, on the other hand, differ considerably. Spectral studies are underway to examine further these differences.

Thermal luminescence from $Er:CaF_2$ has been further studied between $77^{\circ}K$ and $340^{\circ}K$. In particular, two emission lines at 5544 Å and 5585 Å were singled out for study. Using narrow band interference filters coupled to a Fabrey-Perót etalon, it was observed that these lines were emitted in all of the various glow peaks in the above-mentioned temperature range with no detectable shift in wavelength. It was thus concluded that the emitting center for this line was the same for all the various glow peaks.

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7. Stimulated Brillouin Effects in Solids, R. W. Dixon. Contract Nonr-1866(16).

The stimulated Brillouin effect has been observed in quartz crystals as a function of temperature. The apparent threshold power decreases by less than a factor two between room temperature and liquid helium temperature. The crystals always showed material damage, when the stimulated Brillouin effect was observed. Sometimes the stimulated Raman effect was observed in quartz at low temperature, but not in a reproducible manner.

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8. Endor Spectroscopy of Biological Substances, P. Eisenberger. Contract Nonr-1866(16).

A project to study the electron nuclear double resonance signals of hemoglobin and myoglobin-type molecules has begun. Initial efforts have been

directed towards adapting the Endor spectrometer built by J. F. Reichert for electric field effect studies to the problems of interest. A start has also been made to grow single crystals of hemoglobin and myoglobin.

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9. Magneto-optical Properties of Porphyrin-type Molecules, F. J. Kahn. Contract Nonr-1866(16).

Theoretical studies [1] predict significant magneto-optic effects in square porphyrin-type molecules for which the excited states are orbitally degenerate. An experimental program has begun to study these magneto-optic properties. Equipment is currently being assembled to observe the magnetically-induced circular dichroism in these compounds. In cooperation with the Chemistry Department a number of different porphyrin-like molecules are currently being synthesized.

Reference.

1. Fulton, Gouderman, Pershan (in preparation).

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10. Raman Spectra of Solids, C. H. Lee. Contract Nonr-1866(16).

A program of precision measurements on the linewidth and frequencies of the Raman spectra of various inorganic solids has begun. It is hoped to study the temperature and pressure dependents of Raman-active vibration frequencies from these different materials.

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11. Electronic Raman Effect in Solids, G. A. Brooker. Contract Nonr-1866(16).

A program has been initiated to study the electronic Raman effect in a variety of solids. A high quality gas laser will be used for the light source.

Typical samples will initially be rare-earth ions in calcium fluoride. Attempts will be made to study the zero field splittings, magnetic effects, linewidths, and relaxation effects as a function of temperature and pressure for levels not usually accessible by electron spin resonance techniques.

# III. AUTOMATIC CONTROL

## Personnel

Prof. A. E. Bryson, Jr.
Prof. R. E. Kronauer
Asst. Prof. Y. C. Ho
Asst. Prof. K. S. Narendra
Dr. T. S. Baker
Dr. P. G. Drew
Dr. R. J. McLaughlin
Mr. C. R. Arnold
Mr. S. Baron
Mr. C. Blaydon
Mr. M. N. Desai
Mr. R. L. Kashyap
Mr. S. R. McReynolds
Mr. F. Minami
Mr. S. Musa
Mr. C. P. Neuman
Mr. J. L. Speyer

## IIIA. Systems Analysis and Control

1. Adaptive Control, R. E. Kronauer, P. G. Drew, F. Minami. Contract Nonr-1866(16).

Although Dr. Drew completed his work under this contract in June 1964, and is now with Arthur D. Little, Inc., during this period he and R. E. Kronauer prepared a paper on the subject of his previous research for presentation at the International Federation of Automatic Control Symposium, September 1965. The paper deals with the design considerations for the selection of adaptive loop parameters in a parameter perturbation adaptive control system. The notation used represents a considerable refinement over that employed in the thesis and permits a great economy of presentation. New results are obtained which display clearly the design trade-off which exists between rapid adaptation and the residual incremental error when the optimum parameter setting is achieved.

The research of F. Minami on the study of performance criteria suitable for use in adaptive control systems has been completed.

A survey paper on adaptive control by R. E. Kronauer has been completed and accepted for publication by the Institute of Engineers, Australia.

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2. Theory of Oscillations, R. E. Kronauer, R. J. McLaughlin, S. Musa. Contract Nonr-1866(16).

Mr. Musa completed his research on the subject of subharmonic synchronization. This work presents new conditions on the character of nonlinear functions necessary for the synchronization of strong subharmonics of a given order. For pseudo-conservative systems it is shown that the amplitude and phase of the subharmonic solution are related by an integral constraint not previously given in the literature. Also, the problems of subharmonic synchronization in systems known as the Van der Pol and Duffing types are shown to be mathematical duals of each other. These have always been regarded as unrelated. The principle of duality extends to a wide variety of physically interesting systems. This work is now being abstracted into several papers for publication. An additional short paper on integral constraints which exist for coupled oscillations in conservative systems with multiple degrees of freedom has been completed.

The work of Mr. McLaughlin representing an extension of conditions and bounds for the existence of self-excited oscillations is completed. Current efforts are being devoted to development of practical bounds on the accuracy of oscillatory solutions generated by expansion methods. Much work published in the Russian literature is not available in translation and generally proceeds along somewhat different lines from that performed in the U.S.A. It is proposed to compare the different approaches and determine which, if any, is applicable to the transient solution approximation methods.

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## III B. Automatic Control

1. Linear Feedback Solutions for Minimum Effort Interception, Rendezvous, and Soft Landing, A. E. Bryson, Jr. Contract Nonr-1866(16).

The results of this investigation were published in the American Institute of Aeronautics and Astronomy Journal, Vol. 3, No. 8, August 1965, pp. 1542-1544.

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2. A Successive Sweep Method for Solving Optimal Programming Problems, S. R. McReynolds and A. E. Bryson, Jr. Contract Nonr-1866(16).

A preliminary report on this investigation was given at the 6th Joint Automatic Control Conference, Troy, New York in June 1965 (and Cruft Laboratory Technical Report No. 463, March 2, 1965). Mr. McReynolds has nearly completed this work in connection with his dissertation.

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3. Nonlinear Feedback Solution for Minimum-Time Rendezvous with Constant Thrust Acceleration, A. E. Bryson, Jr. Contract Nonr-1866(16).

The results of this investigation were presented at the 16th International Astronautical Congress in Athens, Greece in September 1965, and will be published in the proceedings of the congress. (Also available as Cruft Laboratory Technical Report No. 478, July 15, 1965.)

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4. Nonlinear Feedback Solution for a Brachistochrone Problem with a State Variable Constraint, J. L. Speyer and A. E. Bryson, Jr. Contract Nonr-1866(16).

This work, in preliminary stages, represents one of the four nonlinear feedback solutions for guidance problems with state variable inequality constraints. It uses the dimensionless variable technique developed in (3).

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5. Successive Sweep Applied to Optimal Programming Problems with Inequality Constraints on Control and/or State, J. L. Speyer and A. E. Bryson, Jr. Contract Nonr-1866(16).

This work is an extension of (2) and is completed except for the write-up.

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## III C. Information and Control Processes

### 1. Pattern Classification and Switching Theory, Y. C. Ho* and R. L. Kashyap

R. L. Kashyap has completed his investigation during this period. A summary of results obtained is as follows:

(i) A class of iterative procedures for solving linear inequalities has been developed, and with the aid of these methods the abstraction aspect of any pattern classification problem with known secondary attributes is completely solved.

(ii) Two general methods of designing switching circuits using threshold elements have been developed. These seem to offer significant potential advantages over the existing methods. These techniques are applied to the design of a sine function generator.

Two related papers have been accepted for publication. Y. C. Ho and R. L. Kashyap, "A Class of Iterative Procedures for Linear Inequalities," J. of Math. Anal. and Appl. (to be published), and Y. C. Ho and R. L. Kashyap, "Algorithm for Linear Inequalities With Applications," Trans. of Computer Group IEEE (to be published October, 1965).

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### 2. Pattern Classification, Y. C. Ho and C. Blaydon*

An experiment using pattern classification ideas for the development of feedback controls was investigated (see Cruft Laboratory Technical Report No. 472, "Experiments With a Pattern Classification Technique," by Y. C. Ho, A. Arcese, R. L. Kashyap, and C. Blaydon). An extension of the recent

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*AEC Fellowship

Russian results on pattern classification by Aizerman, Braverman, and Rozonoer have been obtained. (See C. Blaydon, "On a Pattern Classification Result of Aizerman, Braverman, and Rozonoer," IEEE Trans. of Information Theory (to be published October, 1965) ).

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3. Differential Games and Optimal Pursuit-Evasion Strategies, Y. C. Ho and S. Baron*

We are continuing our researches in linear differential games for dynamic systems subject to both energy and amplitude constraints, see Cruft Laboratory Technical Report No. 457, "Differential Games and Optimal Pursuit-Evasion Strategies," IEEE Trans. on Automatic Control (to be published October, 1965).

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*NASA sponsorship

## III D. Topics in Automatic Control

1. Stability of Nonlinear Systems with a Single Monotonic Nonlinearity, K. S. Narendra and C. P. Neuman. Contract Nonr-1866(16).

The investigation of the absolute stability of nonlinear dynamical systems which satisfy neither the Popov nor the extended Popov theorem is being continued. The new Lyapunov function introduced recently by the authors has been applied to derive frequency domain stability criteria for the class of continuous-time systems with a linear plant in the forward path and an odd monotone increasing nonlinearity in the feedback path (Cruft Laboratory Technical Report No. 468). In addition, this same Lyapunov function has been used to present frequency domain stability criteria for the class of discrete-time systems with a linear plant in the forward path and a monotone increasing and an odd monotone increasing feedback function with a continuous and bounded derivative. These frequency domain stability criteria for discrete-time systems are formally identical to their respective counterparts for continuous-time dynamical systems and are applicable to systems whose characteristic equation has some real zeros (Cruft Laboratory Technical Report No. 473). Finally, this new Lyapunov function is currently being applied in order to derive frequency domain stability criteria for continuous and discrete-time systems with m-feedback nonlinearities.

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2. Adaptive Control Using Time Delay and Correlation, K. S. Narendra, T. S. Baker and M. N. Desai. Contract Nonr-1866(16).

The method of adjusting the parameters of a system using time delay and correlation was extended to slowly time-varying situations. The approach

using a model of the system was simulated on an analog computer and a digital computer study of the system using a time delay is in progress. The results will be presented at the Allerton conference in October 1965.

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3. Identification of Nonlinear Systems, C. R. Arnold and K. S. Narendra
   Contract Nonr-1866(16).

The identification of nonlinear systems using iterative and gradient techniques is being investigated. A study of the use of Wiener models in adaptive situations has been carried out and the results will be presented at the Allerton conference.

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# IV. COMMUNICATIONS AND NETWORKS

## Personnel

Asst. Prof. D. W. Tufts
Dr. A. A. Pandiscio
Mr. T. Berger
Mr. H. Gish
Mr. J. Hopkins
Mr. W. Kellogg
Mr. M. Leiter
Mr. J. Proakis
Mr. D. Shnidman
Mr. P. Trafton

## IV A. Communications

1. Transmission Line Distributed Amplifier Using Field Effect Transistors, A. A. Pandiscio and J. Hopkins. Contract Nonr-1866(16).

The system being studied consists of helical input and output transmission lines, coupled by field effect transistors distributed periodically along the lines. A computer program has been prepared which permits the use of realistic circuit models for both the transistors and the loaded transmission lines. The program is a fairly substantial one, as description of the system is complicated by the interaction of various elements. For example, the characteristics of the transmission lines are determined by the input and output impedances of the transistors; and these impedances are affected by the load and generator (output and input line) impedances. Also, the lines are not, in general, perfectly terminated, so each transistor sees a different load and generator impedance. A number of similar considerations are included in the computer simulation, so that it is possible to examine amplifier response as a large number of parameters are varied – transistor characteristics, terminations, impedance levels, etc. As an outgrowth of this, a second, similar program permits study of a cascaded amplifier configuration of any number of stages, allowing different impedance levels for the successive stages.

Results of these programs indicate that the amplifier configuration under examination is a feasible one, which could become competitive for certain applications as better transistors become available. The principal limitation on frequency response is the increase in transistor input conductance as frequency is increased; this tends to reduce the cutoff frequency to about half the value that would be found if decreasing transconductance were the only limitation.

Two amplifiers have been constructed and tested. The transistors used are n-channel depletion-type insulated gate units, which the manufacturer specifies for use up to 60-100 Mc.; the figure of merti $g_m/2\pi C$ is 50 Mc. The first amplifier, designed with these figures in mind, proved to be limited by the line characteristics, which cut off at slightly above 100 Mc. Examination of the transistor equivalent circuit element values showed that the frequency at which the transconductance drops to $1/\sqrt{2}$ times its low frequency value is 400 Mc., and a second amplifier was constructed with line cutoffs of at least 500 Mc. For this case the measured bandwidth was about 200 Mc., in fairly good agreement with the computer simulation. As mentioned earlier, the cutoff is due to attenuation in the input line as the transistor input conductance increases. Voltage gain is limited by the low transconductance of available transistors, and is only about 3 dB. (It should be noted, however, that the optimum stage gain for a cascaded aplifier is only 8.6 dB.)

One important point still to be investigated, principally through computer simulation, is the maximum feedback (gate-drain) capacitance which can be tolerated under various conditions without instability occurring.

It would be useful to be able to use enhancement-type insulated gate transistors in cascaded amplifiers, as they can be designed to operate with the same bias on gate and drain. However, they also have substantial gate-drain capacitance, and this can be particularly troublesome in a distributed amplifier. Similarly, junction field effect transistors are sometimes desirable because of their favorable noise figure and stable characteristics, and these, too, have considerable feedback capacitance. Additional use will be made of the computer simulation to determine optimum configurations and choice of parameters.

Further experimental work is planned, also. The second amplifier was built in such a way as to facilitate changes in the impedance levels, to permit a check on the analytical and computer results which indicate lower cutoff frequencies for higher input line characteristic impedance. At least one other set of impedances will be investigated. Junction (field effect) transistors may also be tried, should the computer simulation indicate their feasibility.

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## IV B. Communications Theory

1. Experimental Investigation of Nonlinear Operations on Random Processes, P. Trafton and D. W. Tufts. Contract Nonr-1866(16).

Work continues on the problems discussed in Cruft Laboratory Technical Report No. 450 and in Semiannual Progress Report No. 69 (July 1, 1964 - January 1, 1965). We have shifted our attention to consider the probability distributions of nonlinear functionals of signal plus noise. From both experimental and theoretical points of view we find that a family of distributions derived from the non-central chi-square distribution provides good approximations.

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2. Joint Optimization of Transmitter and Receiver in Pulse Amplitude Modulation, D. W. Tufts and D. Shnidman. Contract Nonr-1866(16).

Many examples of optimized data transmission were studied in this period to obtain insight into the complicated parameter trade-off relationships in an optimized system. This work is based on the theoretical results contained in Cruft Laboratory Report Nos. 345, 425, and 444.

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3. Optimum Reconstruction and Random Waveforms from Quantized Signals, W. Kellogg and D. W. Tufts. Contract Nonr-1866(16).

A complex computer program is being written to compute optimum combinations of prefilters and postfilters for the processing of quantized samples of a random waveform. The least mean square reproduction of the waveform is our basic goal.

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4. Bounds on Channel Performance, T. Berger. Contract Nonr-1866(16).

Work continues on the calculation of performance bounds for realistic communication channels with memory, using Shannon's theory of Rate-Distortion functions.

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5. Error Probability Comparison of PSK Signaling Systems, M. Leiter. Contract Nonr-1866(16).

Heretofore intuitive design philosophies for Phase Shift Keying data communications systems are being evaluated by means of a detailed error probability analysis.

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6. Adaptive Transmitters and Receivers for Nonstationary Message Sources, D. W. Tufts and H. Gish. Contract Nonr-1866(16).

Transmitters and receivers which adjust appropriate parameters in response to measurements on an input process are being considered both within a general theoretical framework and within the practical constraints of a Delta Modulation system.

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7. Signal Detection Using Noisy Reference Signals, J. Proakis. Contract Nonr-1866(16).

Work has continued on the problem of signal discrimination and signal parameter estimation when the discrimination and estimation procedures are tied together. Some recent results have been published in the Cruft Laboratory Technical Report No. 465, which is dated April 29, 1965.

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# V. MICROWAVE APPLICATION OF FERROMAGNETIC AND FERROELECTRIC MATERIALS

## Personnel

Prof. R. V. Jones
Mr. J. Comly
Mr. B. Levine
Mr. F. Milton
Mr. T. Penney
Mr. R. Tancrell

Contracts Nonr-1866(16) and AF19(628)-3874.

In a parallel pumping experiment, a microwave magnetic pump field is applied parallel to the dc bias field in a magnetic sample. The spin precession is elliptical in many of the magnetic excited states, spin waves, giving rise to a moment along the bias field at twice the spin wave frequency. The pump field couples power into pairs of spin waves at half the pump frequency, to conserve energy, through this longitudinal moment. When the power pumped into the spin wave exceeds the power out through the thermal relaxation channel, the spin wave goes unstable and grows very large. Its amplitude is eventually limited by nonlinear interactions with other spin modes. Spin waves of widely varying wavelength may be driven this way. The threshold for this parallel-pumped instability (PPI) can be observed as a sudden power-dependent increase in the microwave sample loss. The pump field for threshold is a measure of the relaxation rate of the spin wave with the lowest threshold at a particular bias field and frequency.

Detailed understanding of some of the fundamental relaxation processes in ferrimagnetic insulators has been gained through the study of PPI. Similar experiments would be useful in ferromagnetic metals. Unfortunately, the active sample volume is limited in metal samples because of the microwave skin depth which makes signals small. A magnetron must be used to supply the high power which is needed to reach the threshold pump field. The combination of these

experimental difficulties makes it hard to observe PPI in metal ferromagnets. The observations reported in this work were made with a pulse-power modulation analog of the usual EPR field modulation signal recovery technique. They were made on vacuum-deposited films and rolled foils of the nickel: iron alloy, Permalloy, and are the first useful measurements of PPI thresholds in ferromagnetic metals.

The most important result of this study is that PPI signals can be observed both in films which are thinner than a microwave skin depth, and in polycrystalline foils which are many skin depths thick. There had been some question about this possibility before these measurements were made. Analysis of the PPI thresholds gives relaxation rates which are in excellent agreement with resonance linewidths in situations where the resonance linewidths should not be affected by two-magnon inhomogeneous broadening. This PPI determination of the relaxation rate is insensitive to inhomogeneous broadening effects and does not rely on the measurement of any resonance linewidth.

The intrinsic relaxation time of 83:17 Permalloy films is about $1.4 \times 10^{-9}$ sec, which is represented by a linewidth of 80 Oe. A rough extrapolation of our alloy data is in agreement with Rodbell's resonance linewidth measurements in single crystal platelets of nickel, within the limits of our extrapolation error. The agreement in these two cases shows that two-magnon inhomogeneity scattering does not constitute the largest part of the spin wave resonance linewidth in thin films of Permalloy, or in good single crystals of nickel. The fundamental relaxation processes which cause these linewidths have yet to be identified. Several possibilities are discussed.

Another aspect of our results concerns some details of the magnetic excited states which exist in a thin ferromagnetic sample. An oscillatory struc-

ture occurs in thick films in the dc field dependence of the loss susceptibility above PPI threshold. This structure yields information about the magnetic excited states which exist in regions where the sample shape demagnetization and exchange forces are of equal importance. Apparently, the spin modes in a thin film have frequencies which are the same as in bulk insulators for the same wave vector, k. However, the wave vector across the film thickness, $k_x$, is quantized in units of $\pi$ /(film thickness) which leads to a change in the direction of propagation of the spin wave with the lowest threshold as the bias field is varied. The propagation direction varies periodically with field which leads to periodic PPI coupling changes. These coupling changes lead, in turn, to oscillations in the signal at constant power above the minimum threshold power.

Very thin films, less than 400 Å thick, show susceptibility structure which reflects the fact that even the shortest wavelength spin waves that can be excited by the pump field are still longer than the sample thickness. As a result, all spin waves see about the same demagnetization field as the uniform mode. In this situation, the bulk spin wave manifold is not at all an accurate picture of the true spin modes. A qualitative picture of the correct modes in a very thin film, along with the thin film loss susceptibility data, allows the basic mechanism which limits the spin mode amplitude above PPI threshold to be determined unambiguously.

# VI. ELECTROMAGNETIC RADIATION

## Personnel

Prof. R. W. P. King
Assoc. Prof. T. T. Wu
Asst. Prof. B. Rama Rao
Dr. K. Iizuka
Dr. S. S. Sandler
Dr. S. R. Seshadri
Dr. H. -S. Tuan
Mrs. D. G. Tingley
Mr. D. C. Chang
Mr. C. -L. Chen
Mr. W. -M. Cheong
Mr. B. M. Duff
Mr. S. Holly
Mr. D. Lamensdorf
Mr. R. J. Mailloux
Mr. I. L. Morris
Mr. T. Padhi
Mr. R. D. Ruquist
Mr. W. A. Saxton
Mr. L. -C. Shen
Mr. R. Shore
Mr. T. Sugimoto
Mr. C. -Y. Ting
Mr. A. D. Wunsch

1. Electromagnetic Scattering by a Conducting Cylinder Coated with an Anisotropic Ferrite or Plasma Sheath - Theoretical and Experimental Studies, B. Rama Rao. AF19(628)-2406.

A theoretical study is being made of the scattering of electromagnetic waves by an infinitely long conducting cylinder coated with an anisotropic ferrite sheath magnetized along its axis. A harmonic series solution for the scattered field has been obtained both for the case of an incident plane wave with electric vector polarized parallel to the cylinder and for a unit electric line-source parallel to the cylinder. The effect of the magnetostatic field on the back-scattering cross section and the total diffracted fields in the illuminated and shadow regions have been investigated. Numerical results have been obtained with an IBM 7094 computer. To substantiate the theory, experiments are being made in a parallel-plate transmission line using an Indiana General R-6 Mg-Mn-Al Ferrite sample.

Experiments will also be made shortly on electromagnetic scattering from plasma-coated metallic cylinders. The plasma column will be generated by means of a brush-cathode-type discharge tube.

The scattering from a large plasma-coated cylinder is being studied using geometrical optics techniques.

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2. Antenna Theory and Wave Theory of Long Yagi-Uda Arrays, R. J. Mailloux. National Science Foundation Grant NSF GP-851.

The expressed intention of this research has been to investigate the possibility of developing a wave theory for long Yagi arrays which is consistent with the accepted antenna-array theory. The practical significance of this study is derived from the fact that conventional array theory demands the inversion of large matrices, and so becomes intractable when the number of dipoles in the array becomes sufficiently large. On the other hand, a wave theory solution could be adapted to nearly any length array once a certain minimum length is exceeded.

The program of research was divided into three main topics: a comparison of wave and antenna theories, a study of the excitation of a wave on an infinite array, and an experimental and analytical study of the finite Yagi array. The first topic is described in a report, "A Unification of Antenna Theory and Wave Theory: Infinite Yagi-Uda Arrays", Cruft Laboratory Technical Report No. 451, June 22, 1964, and a summary of this work has been accepted for publication by the IEEE Transactions on Antennas and Propagation and will appear in the July 1965 issue. In this report a wave analysis is formulated which is consistent with antenna theory, and then this new concept is applied to consider some parameters of Yagi behavior.

The second topic is also discussed in a technical report: "The Excitation of a Surface Wave Along an Infinite Yagi-Uda Array", Cruft Laboratory

Technical Report No. 456, October 12, 1964, and a summary and extension of this work has been accepted for publication in the September 1965 issue of the IEEE Transactions on Antennas and Propagation. This problem was of particular interest because it showed for the first time that a wave could be excited on an infinite array, and it showed the relative importance of this contribution to the input admittance and the current distributions in the dipoles. The range of dipole heights where the surface wave ceases to exist (stop band) is studied in some detail in the paper because it is of great fundamental importance in demonstrating the validity of the wave concept for long finite arrays. This intersection of the stop and pass bands is henceforth referred to as the "critical point".

The third topic considered has been the long, finite Yagi array. This subject provides a convenient vehicle for testing the validity of the King-Sandler theory and the wave theory, and for observing effects in the array solution which are inherently related to the wave properties. Several twenty-element Yagi arrays of various dipole heights were studied experimentally, and current and charges in the various dipoles of the arrays were recorded. Input admittances and array field patterns were also recorded. These experimental results were compared first with the King-Sandler theory and then with wave theory (adapted for finite arrays). In both cases the agreement was excellent. This material is described in Cruft Laboratory Technical Report No. 464, March 30, 1965.

The "critical point" for the surface wave solution was investigated by comparing the array solution and the wave solution, and it was shown that the wave properties enter dramatically into the King-Sandler solution. This fact

thus provides a final contribution to the evidence which links the wave theory to the accepted array theory.

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3. The Cylindrical Antenna with Arbitrary Driving Point, R. W. P. King and T. T. Wu. Contract Nonr 1866(32).

The approximate three-term formula for the distribution of current in a cylindrical antenna has been generalized to provide a new and more accurate solution of the asymmetrically driven antenna. Such a solution is of value for antennas which have active elements distributed along their length. A paper entitled "The Cylindrical Antenna with Arbitrary Driving Point" by Ronold W. P. King and Tai Tsun Wu will be published in the September 1965 issue of the IEEE Transactions on Antennas and Propagation. Technical Report No. 455 on this subject has been issued.

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4. Optimization of Curtain Arrays, I. L. Morris. National Science Foundation Grant GP-851.

Ever since the Yagi array was conceived it has aroused much interest due to its simple construction and the possibility of modifying its properties by varying the different parameters (length, radius, and spacing of the elements). The problem of optimizing a given design with respect to such criteria as gain, front-to-back ratio or input impedance necessarily requires the consideration of a great many combinations of these parameters. Furthermore, it requires an adequate theoretical solution of the Yagi array which does not ignore the coupling between the elements.

Early investigators incorrectly assumed sinusoidal current distributions on the elements, which does ignore the coupling between them. Others made

the additional assumption (also incorrect) that the director currents have the same amplitude and phase delay. A theoretical solution of the Yagi array has been found which involves a numerical solution of the simultaneous integral equations relating current distributions to vector potentials. It is based on the fact that a modification of the quasi-zero-order theory introduced by King in connection with circular arrays and later applied to curtain arrays by Sandler is applicable to the Yagi array. Coupling between the elements is not neglected and the current distributions are not assumed to be identical.

The modified quasi-zero-order theory incorporates approximations to the integrals that are independent of element length and spacing. Besides being easier to program for high-speed computation, the modified theory yields solutions which in many instances satisfy the integral equations better than the original quasi-zero-brder solutions. The agreement between solutions of the modified theory and experimental results is very good. Comparisons have been made for currents, residuals and radiation patterns for 4- and 10-element Yagi arrays.

The second part of the investigation was concerned with the optimization of a Yagi array with directors of constant length, radius, and spacing. The computer programs used for this purpose were written in FORTRAN for the IBM 7094 computer and employ the modified quasi-zero-order theory. Since a great many configurations had to be investigated, great care was taken to reduce the execution time and facilitate the reduction of the data. The former was accomplished by careful design of the programs, the latter by selecting a convenient output format. For a given (N-2)-director Yagi array, there are only two parameters (director length and spacing) which are varied. By printing out the data in columns and rows, which are spaced in accordance with the increments in director spacing and

length, it is possible to construct manually contours of constant gain, etc., directly on the printout. These maps serve to summarize compactly the detailed investigation of the Yagi array and play an important role in the search for optimum designs.

Detailed investigations of the 1-and 4-director Yagi arrays have been summarized in the form of contour maps for forward gain, backward gain, front-to-back ratio, input resistance, and input reactance. In addition, some of the optimum designs (with respect to forward gain or front-to-back ratio) for the 1-, 2-, and 4-director arrays have been compared. Finally, based upon the investigations of the 1-, 2-, and 4-director Yagi arrays, certain characteristics of the 8-director array have been predicted and verified.

This investigation has been completed and two Scientific Reports have been issued: "Optimization of the Yagi Array I", S.R. No. 6 (Series 3) and "II", S.R. No. 10 (Series 3).

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5. Theoretical Study of the Resistive Antenna, R. W. P. King and T. T. Wu. Contract Nonr 1866(32) and NASA Grant NsG-579.

Machine computations of current distributions and impedances of resistive antennas for a wide range of values of the resistance per unit length have been carried out at the Sandia Corporation. A joint paper extending the work in Technical Report No. 440 will be prepared.

A paper entitled "The Cylindrical Antenna with Non-Reflecting Resistive Loading" by T. T. Wu and R. W. P. King has been published in the May issue of the IEEE Transactions on Antennas and Propagation.

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6. Theoretical and Experimental Studies of the Resistive Antenna, L.-C. Shen. NASA Grant NsG-579.

A. Theory. The current, the input impedance and the far field pattern of a cylindrical antenna with resistive loading have been determined theoretically. The distribution of the resistive loading along the antenna is a particular function multiplied by a real constant parameter $\alpha$. The current on the antenna and the field pattern have been evaluated for a wide range of lengths with several different $\alpha$'s ranging from 0 to 1 and for positive integers. They have been found not critically dependent on the parameter $\alpha$. For $\alpha$ near or greater than 1, the antenna is non-reflecting. A report on this study is being prepared.

B. Experiment. The purpose of the experiment is first to construct an antenna with resistive loading prescribed by the function mentioned in A and then to compare the measurements of the current distribution and the far field pattern with the theory. The resistive loading is to make use of resistive paint sprayed over a dielectric rod. It has been found that the internal resistance required to realize the antenna can be obtained by either changing the thickness of the coating or the content of the resistive paint. This experiment is in progress.

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7. Experimental Study of Electrically Thick Antennas, S. Holly. Contract AF19(628)-2406.

The apparatus which was built previously to measure the characteristics of electrically thick antennas was used in the past reporting period to measure the input admittances of these antennas. The apparatus consists of 3 antenna-coaxial line sets with different diameters (antenna radii: $a_1$ = 2.54cm; $a_2$ = 5.08cm; $a_3$ = 6.985cm) which can be mounted on a large outdoor groundplane. Antenna heights are continuously variable for all 3 dif-

ferent diameter sets between 0 and 50cm. Input admittance values of these 3 antennas were obtained as a function of antenna height at 8 different frequencies ranging between 461.5 Mc and 1 Kmc. The admittance measurements were made with the top of the antenna tubes either open, closed with a flat metal disk, or covered with a hemispherical cap of the same diameter as the antenna tube itself. The admittance values were measured on a slotted line built right on the outer conductor of the antenna feeding coaxial line, by using the 3 db point method. The antenna feed coaxial line was transmitting the RF signal to the antenna in the conventional TEM mode. However, since the cutoff frequencies for the first higher order i.e., $TE_{10}$ mode for the 3 different diameter coaxial lines were 1694.6 Mc, 847.3 Mc and 616.2 Mc respectively, above or near these cutoff frequencies the presence of this higher mode could be observed together with the TEM mode. Attempts to establish uniform angular field distribution in the coaxial lines at these frequencies were not successful.

A computer program was written for reducing the measured data to impedance and admittance values. These results have been tabulated for those 17 frequencies at which all the higher order coaxial modes are below cutoff. Some typical sets of values have been plotted. These results will be presented in a Scientific Report in the future.

At present, experiments are in progress for measuring the relative magnitudes and phases of the axial distributions of current and charge along these 3 different diameter antenna sets.

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8. Theory of the Thin Circular Loop Antenna, R. W. P. King, D. G. Tingley, T. T. Wu, and C. W. Harrison, Jr. Nonr 1866(32).

A paper entitled "The Current in Bare Circular Loop Antennas in a Dissipative Medium" by R. W. P. King, C. W. Harrison, Jr. and D. G. Tingley was published as a Communication in the July 1965 issue of the IEEE Transactions on Antennas and Propagation. This supplements the earlier paper "The Admittance of Bare Circular Loop Antennas in a Dissipative Medium" which appeared in the July 1964 issue of these Transactions.

A chapter on the "Loop Antenna for Transmission and Reception" has been prepared by R. W. P. King for a book on Antenna Theory edited by R. E. Collin and F. J. Zucker. The chapter summarizes recent work on the loop prepared primarily under this Contract.

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9. Theory of Coupled Long Antennas, T. Padhi. Contract Nonr 1866(26) and the Sandia Corporation.

A good mathematical model for an antenna geometry that has been used in some experimental studies of transients is a semi-infinite linear antenna parallel to an infinite one with a delta-function generator. In most cases of interest, this model would predict correctly the current on the parasite until the appearance of multiple reflections. This problem can be formulated in terms of a Wiener-Hopf equation and so may be solved in principle. However, the expressions obtained so far are too involved for direct numerical computation of results and considerable simplification is needed to put them into computable form.

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10. A Study of Curtain Arrays of Dipole Antennas, S. S. Sandler, R. B. Mack and R. W. P. King. AF 19(628)-2406.

Three of seven chapters have been completed in first draft for a monograph that is to provide a coordinated, relatively elementary summary of array theory based on the work of King, Mack, Sandler and Morris.

A chapter on "Cylindrical Antennas and Arrays", in which an integrated rather advanced summary of array theory based on the work of King, Mack and Sandler is presented, has been prepared by R. W. P. King for a book on Antenna Theory under the editorship of Prof. R. E. Collin and F. J. Zucker.

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11. The Biconical Antenna in a Radially Stratified Medium, J. Fikioris. Contract Nonr 1866 (32).

A paper entitled "Asymptotic Expansion of Solutions of Differential Equations" by J. G. Fikioris has been published in the Journal of Mathematical Physics, Vol. 6, No. 7, 1131-1148, July 1965.

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12. A Study of Circular Antenna Arrays, R. B. Mack. Contract Nonr 1866(32).

Several papers for publication in the scientific literature are being prepared from the Cruft Laboratory Technical Reports Nos. 381-386 issued on completion of this project.

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13. Propagation of Electromagnetic Waves in an Acoustically Disturbed Plasma, W. A. Saxton. National Science Foundation Grant-2242.

Initial results have been reported in an experimental study of acoustic disturbances in weakly ionized gaseous plasmas which exist in the positive

column of dc discharges. One phase of the research concerned itself with the effects of externally perturbing such a medium with acoustic waves emanating from transducers that operate in the audio and ultrasonic frequency ranges. The balance of the effort was devoted to the observation of sound waves generated by natural and uncontrolled low-frequency oscillations which commonly occur in laboratory discharges.

Two compact transducers, which are incorporated into cylindrical plasma-discharge tubes, are the sources of acoustic signals and serve as microphones in detecting sound waves. Although most of the data were taken with a modified bipolar moving-armature device which operates in the low audio range, a solid-dielectric-type transducer with potential application at ultrasonic frequencies has also been investigated.

Modulation of electromagnetic waves by intentionally disturbed plasmas was measured in a novel rectangular cavity whose output was detected and fed to a wave analyzer. Resultant wave-analyzer responses indicate that the electron collision frequency was modulated in addition to the electron plasma frequency, and that the variations in both were proportional to the magnitude of loudspeaker diaphragm deflection, as predicted by simple acoustic theory. Knowing that two forms of modulation occur simultaneously, a calibration scheme was developed to separate the variation in the plasma frequency from the total response. Measurements to date indicate that $\Delta N_e/N_e = \Delta N/N$ (where $N_e$ and N are electron and neutral molecule densities, respectively) for slightly ionized gases when subjected to low-frequency pressure variations in the order of $10^{-5}$ Torr at ambient equilibrium pressures between 100$\mu$ and 300$\mu$, and that such variations produced plasma-frequency pertur-

bations of 0-140 kc/s at plasma frequencies up to 1000 Mc/s.

Early observations of intentional acoustic perturbations were made in stable and quiescent discharges over pressure ranges for which large spontaneous oscillations in the positive column were absent. However, frequent and unpredictable appearance and disappearance of moving striations made it extremely difficult to maintain a quiet plasma and, as measurements progressed, there were indications that the medium was often in a quasi-stable condition at which time the sound waves transmitted into the positive column of the discharge by the transducer actually excited a low-frequency oscillation and interacted with it. Ultimately, a separate investigation was conducted to determine the nature of the coupling between the spontaneous plasma oscillations and the sound waves generated by the transducer. In addition to an attentuation of the sound waves propagating into the plasma, a clear-cut frequency-mixing process was observed between the low-frequency oscillations and the forced sound waves.

Finally, the transducer was used as a microphone without the RF circuitry to "listen" to the sound waves created by the moving striations in the neutral gas. Although it was impossible to correlate the magnitude of the low-frequency oscillations with the intensity of the sound waves produced by them, a one-to-one frequency correspondence was in evidence at all times. Thus, a link was established between the electrical oscillations in the plasma and acoustic waves in the neutral gas; and, a technique was demonstrated for quantitatively investigating the relationship between the two.

Two technical reports on this research will be issued soon.

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14. Wave Propagation in Anisotropic Media and in Plasmas, H. S. Tuan and S. R. Seshadri. Nonr 1866(32).

A paper entitled, "Radiation from a Phased Line Source in a Magneto-ionic Medium," by H. S. Tuan and S. R. Seshadri has been accepted for publication in the Canadian Journal of Physics.

The radiation resistance of a circular filament of electric current immersed in an unbounded magneto-ionic medium has been evaluated for the case in which the axis of the loop is parallel to the direction of the external static magnetic field. Numerical values of the radiation resistance have been obtained for two different sizes of the loop and for some typical values of the plasma parameters, for the special case of the dominant mode excitation for which the current is a constant around the loop. A technical report summarizing the results of this research is being written up.

Radiation from a prescribed electric current on a filamentary loop in a warm, unbounded plasma has been investigated. The formula for radiation resistance is obtained in the general case. A numerical example is obtained specifically for the case of radiation from the distribution of a dipole mode of the current on the loop. A technical report is in preparation.

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15. Currents, Charges and Near Fields of Radiating Elements, R. W. P. King and T. T. Wu. Contract Nonr 1866(32).

A paper entitled "Currents, Charges and Near Fields of Cylindrical Antennas" by R. W. P. King and T. T. Wu was published in the April, 1965 issue of Radio Science. A communication entitled "Currents, Charges and Near Fields of Cylindrical Receiving and Scattering Antennas" by R. W. P. King and T. T. Wu has been accepted for publication by the Transactions

of the IEEE, GAP.

The calculation of the electric field tangent to and very near a cylindrical antenna involves not only the current distribution but also the slope of the charge distribution. Since this latter is not well represented in general by the simple three-term formula derived in the earlier papers, a more accurate theory for this component of the field was required. A first-order current has been obtained by a single iteration of the previous theory for both transmitting and receiving antennas. The tangential electric field computed from these improved currents differs greatly from the zero-order values previously obtained except in the case of a full wave scattering antenna for which the zero-order approximation is quite good. A short paper supplementing the earlier paper has been submitted to Radio Science.

In the case of the scattering antenna, an alternative approach has been developed for determining the integral equation for the tangential component of the electric field near the antenna. A zero-order solution of this equation yields a surprisingly simple expression for the field which is in excellent agreement with the rather complicated results obtained with the first-order current. A paper on this subject has been submitted to the Institution of Electrical Engineers, London.

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16. A Study of the Junction Between Perfect and Imperfect Conductors in a Coaxial Line by the Wiener-Hopf Technique, R. D. Ruquist. Contract AF19 (628)-2406 and NASA Grant NsG-579.

The current distribution near the junction in a coaxial line of a perfectly conducting inner conductor with an inner conductor formed of a resistive coating on a dielectric rod has been determined together with the associated

transmission-line parameter. The problem has been solved by a Wiener-Hopf technique for electromagnetic waves incident from both the perfectly conducting side and from the lossy side. The complex transcendental kernel of the Wiener-Hopf equation has been factored by four methods, one of which is rigorous. The three approximate methods are conventional. The rigorous technique expresses the kernel in the form of a product expansion. A new phase-locus method was used to solve two-dimensional transcendental equations for the poles and zeros of the expansion. The results indicate that conventional transmission-line theory and the approximate techniques of factorization are in error when the resistance per unit length of the resistive coating exceeds the characteristic impedance of the transmission line. Supporting experimental data have been taken.

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17. An Experimental Study of the Properties of Antennas when Immersed in a Conducting Dielectric, K. Iizuka and T. Sugimoto. Nonr 1866(32).

A large wooden tank (approximately 400 gals.) has been constructed to contain a jelled solution of agar agar. By diffusing either a salt solution or alcohol through the surface of the agar, a medium with a gradient in either conductivity or dielectric constant can be obtained. This permits a close simulation of several interesting naturally occurring environments (ionosphere, plasma sheathes, ocean). The properties of the agar agar with and without the addition of a salt solution are presently being studied prior to the actual measurement of properties of antennas immersed in the agar. The investigation is being performed in the frequency range from d.c. to microwave frequencies.

An experimental determination of the reflection off the plane surface of a lossy medium for different angles of incidence is also being carried out. The transmitting and receiving antennas are mounted on a supporting arc above the agar tank and measurements are made at X-band frequencies.

An experimental study has been made of coupled dipoles in a dissipative medium. Input impedances have been measured for selected lengths and spacings and compared with an extensive set of theoretical values based on the theory of R. W. P. King and computed at the Sandia Corporation under the direction of C. W. Harrison, Jr. A technical report containing both theoretical and experimental results is in preparation.

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18. Studies of Loaded Loop Antennas, K. Iizuka. Contract Nonr-1866(32).

Studies on arrays of circular loops which sustain the propagation of slow waves are continued.

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19. Traveling-Wave V-Antenna, K. Iizuka and R. W. P. King NASA Grant NsG-579.

An antenna which is a combination of the resonant V-antenna and the traveling-wave dipole antenna was studied both theoretically and experimentally. Universal curves which are useful for determining the length of the antenna arms, the apex angle, and the loading resistors are presented. The theoretical curves for the radiation pattern agree in a general sense with those of the experiments. The antenna has a pencil-beam radiation pattern. The two-wavelength long traveling-wave V-antenna has a gain 10.5 times as large as that of a half-wave dipole. Scientific Report No. 4 entitled "The Traveling-Wave V-Antenna", by K. Iizuka and R. W. P. King has been issued.

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20. Theoretical Study of Antennas in Plasmas, A. D. Wunsch. NASA Grant NsG-579.

Two theoretical investigations involving antennas immersed in plasmas are being carried out.

The problem of finding the current distribution on a dipole antenna surrounded by a homogeneous, compressible (finite temperature) plasma is close to solution. A method has been found for obtaining the coefficients of a Fourier series used to describe the current. An examination of the validity of the approximations involved in obtaining these coefficients is now in progress.

A letter commenting on an existing solution of the problem of finding the impedance of such a plasma-immersed antenna has been accepted for publication by the Proceedings of the Institution of Electrical Engineers. In addition, an unpublished note of the author is described and acknowledged in a book, "Antenna Theory", to be published by McGraw-Hill (edited by R. E. Collin and F. J. Zucker). The note is concerned with the surface-wave current existing on an infinite antenna immersed in a compressible plasma, and is mentioned in a chapter on antennas in plasma by James R. Wait.

In another investigation, recently begun, the radiation resistance of an electrically short antenna immersed in a cold magneto-plasma is being determined. In this analysis an assumed current distribution is used. Particular attention is being given to the case in which the static magnetic field of the plasma is oriented normal to the axis of the antenna. The method of solution involved here is one recently proposed by S. R. Seshadri.

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21. Antenna in a Cylinder of Dissipative Material, D. Lamensdorf. National Science Foundation Grant GP--2242.

Measurements are being made of the admittance and the current distribution of a monopole antenna surrounded by a cylinder of lossless dielectric. Parameters varied include the dielectric constant, cylinder diameter and length, and antenna length. The length of dielectric cylinder needed to approximate an infinite cylinder for a given set of the other four parameters has been determined empirically. The possibility of scaling relationships between the antenna characteristics for cylinders of finite diameter and infinite diameter is being examined.

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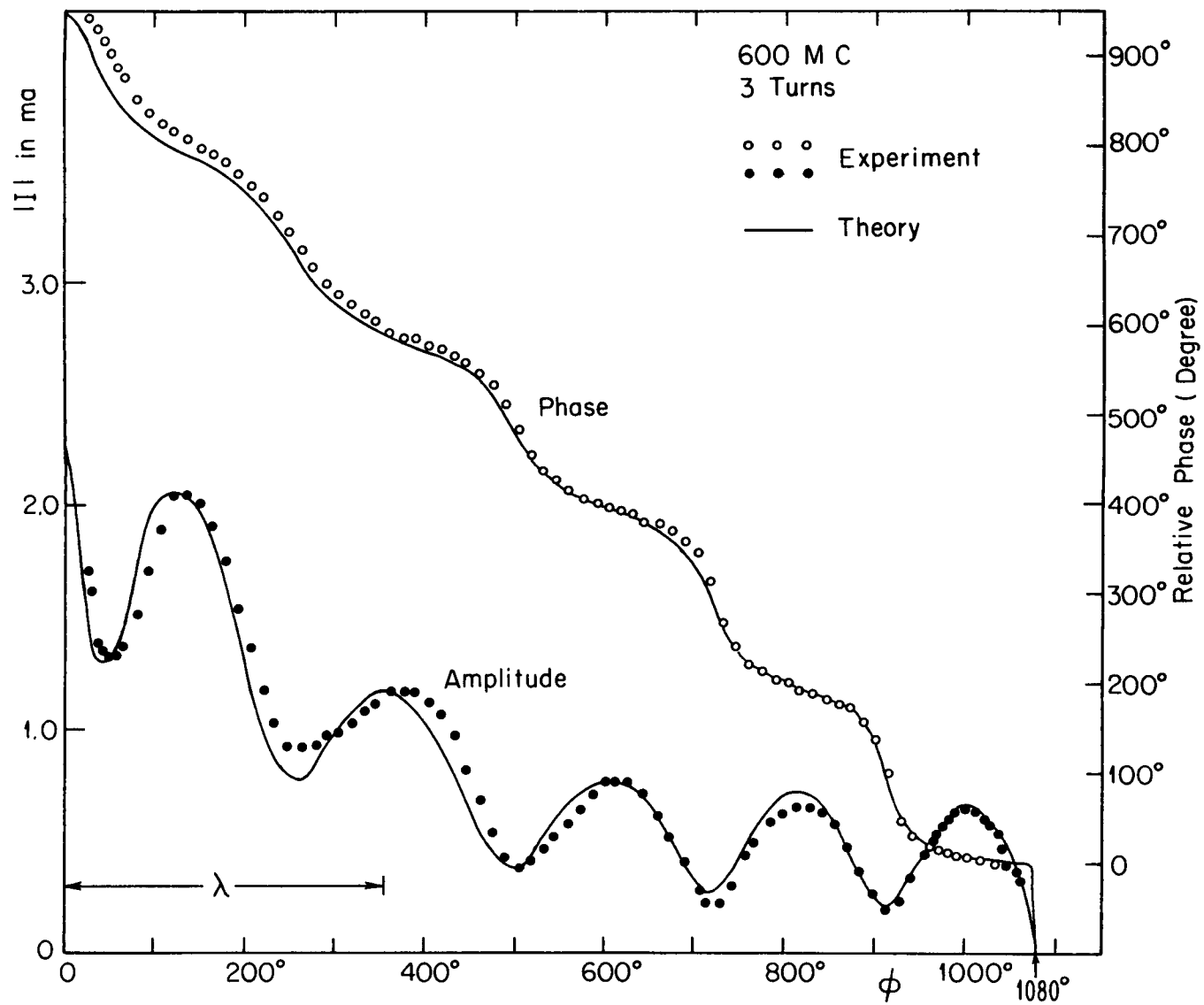
22. A Theoretical and Experimental Study of Helical-Wire Antennas, C.-L. Chen. Contract AF19(628)-2406.

A finite, balanced, helical-wire antenna excited by a delta-function generator has been investigated. The geometry of a helical wire has been examined carefully. Based on this analysis an exact integral equation for the current distribution along the antenna is formulated. For a thin helical wire, the exact integral equation can be replaced by a one-dimensional approximation, which can be solved by a Fourier series method. An approximate solution for the current distribution was then obtained in conjunction with the logarithmic singularity. The logarithmic singularity is subsequently subtracted out by comparing the Fourier series solution for the dipole antenna with Wu's solution. Thus the input admittance was obtained.

Experiments were carried out to measure the current distribution along and the input admittance of the balanced helical-wire antenna. The ac-




EXPERIMENTAL AND THEORETICAL CURRENT DISTRIBUTION

companying figure shows the experimental points together with the theoretical curve. The theoretical analysis was thereby confirmed experimentally.

Based on this investigation the following conclusions can be drawn:

1. The current distribution in the vicinity of the driving point, and hence the input admittance, are insensitive to the actual length of the antenna.
2. The current decays away from the feed point exponentially.

Since this exponentially decaying behavior is quite different from the usual assumption of a uniform traveling-wave current distribution along a helical-beam antenna, an investigation of the radiation pattern of a balanced helical-wire antenna will be very instructive and is in progress.

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23. The Shielding Against Transient Electromagnetic Fields by Imperfectly Conducting Cavities, R. W. P. King and T. T. Wu. The Sandia Corporation.

The following paper has been published: "The Propagation of Transient Electromagnetic Fields into a Cavity Formed by Two Imperfectly Conducting Sheets," C. W. Harrison, Jr., M. L. Houston, R. W. P. King, and T. T. Wu, IEEE Transactions on Antennas and Propagation, AP-13, No. 1, January 1965.

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24. Experimental Study of Two Parallel Circular Arrays and of Two Parallel Electrically Thick Antennas, B. M. Duff. National Science Foundation Grant NSF GK-273.

Preparations are being made for the experimental measurement of the magnitude and phase of current and charge distributions, and of impedances of two types of antenna arrays. The first array to be studied consists of two

parallel circular arrays of thin elements. Each circular array will consist of four elements spaced on the circumference of a circle of 2.00 in., 4.00 in. or 5.50 in. diameter. At an operating frequency of 600mc/s these diameters correspond to 0.1016λ, 0.2032λ and 0.2781λ. A number of different spacings between centers of the two circular arrays will be used. The second array consists of two parallel electrically thick antennas. The diameters of the antennas and spacing between centers will be identical to the corresponding dimensions used in the study of circular arrays.

Most of the apparatus for these experiments has been constructed, including the three different sizes of thick antennas and associated coaxial-line driving systems. The two circular arrays have been assembled on a small indoor groundplane where preliminary measurements are now in progress. Final measurements on both systems will be made on a large outdoor groundplane.

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25. Circular Arrays with Elements of Large Radius, D. Chang. Contract Nonr 1866(32).

The problem of N-element circular arrays with large radius (ka = 0.1-0.7) is solved by assuming King's three-term theory, i.e., the real part of the current is assumed to be composed of sine and shifted cosine terms while the imaginary part is a shifted half cosine. Numerical calculations of current and impedance have been obtained for arrays of up to 64 elements. As N is increased, it is noticed that the total power radiated by the array converges to a certain value, which indicates the validity of using the array model to calculate the impedance of a thick antenna.

Comparison has been made between the experimental data for a solid thick antenna above a ground plane and fed by a coaxial line and the numerical

calculations of the array model. Over a range of antenna lengths, the input conductance obtained from the two methods is in reasonably good agreement, but further investigation is needed on the correction for the input susceptance.

The problem of two coupled thick antennas is also under investigation.

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26. Slot Transmission Lines and Radiators in Nonplanar Structures, R. W. Burton. Contract Nonr 1866(32).

A paper entitled "Two-Slot Line on Nonplanar Surfaces" by R. W. Burton and R. W. P. King has been published in the IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-13, No. 3, May 1965.

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27. Theoretical and Experimental Studies of Log-Periodic Antennas, W.-M. Cheong. NSF-GK-273.

The log-periodic antenna with coaxial feed is being constructed in the machine shop and the groundplane and supporting blocks for the measuring coaxial lines are being readied for the experimental measurements. A computer program is being written to facilitate the theoretical analysis of the log-periodic structure.

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28. Two-Wire Line Immersed in a Low-Density Plasma, T. Padhi. Contract Nonr 1866(26) and the Sandia Corporation.

A study of the characteristics of a two-wire line immersed in a low-density plasma is being made to determine whether it offers a suitable technique for the measurement of the propagation constant in the plasma over a wide frequency range. This technique may be applied to establish experimentally, or disprove, the presence of a "window" below the plasma fre-

quency when the plasma is disturbed by a strong ion-acoustic wave. Both propagation constant and impedance of the two-wire line depend in a complicated way on the d.c. bias applied to the line. The theory necessary to relate experimental measurements to plasma parameters is at present in a rather primitive state and so is also being studied.

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29. Design and Study of a Broadband, Traveling-Wave Receiving Dipole Antenna, R. D. Ruquist. NASA Grant NsG-579.

A new solution has been obtained for the optimum design of broadband reflectionless traveling-wave-dipole receiving antennas when treated in the zero-order-transmission-line approximation. A reflectionless traveling-wave-current distribution is postulated and its differential equation determined. The current distribution is not physically realizable with only a variable resistance coating on the inner conductor of the coaxial line or the antenna because the differential equation contains a frequency independent coefficient. A trial-and-error method was used to select the current distribution that minimizes the reflection coefficient. The reflection coefficient was calculated numerically for each case. Supporting experimental data have been taken.

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30. Antenna in Conducting Half-Space, H. S. Tuan. Contract Nonr 1866(32).

An approximate current distribution has been obtained for a half-wavelength antenna immersed in a conducting half-space excited by a diffracted inhomogeneous plane wave. The distribution is found to be mainly cosinusoidal with a shifted cosine correction term. A numerical example has been calculated for the case of a half-space with constants approximated by average values for the earth.